A

Test Registration No.: __________

The Maharaja Sayajirao University of Baroda

Faculty of Science

M. Sc. ENTRANCE EXAMINATION

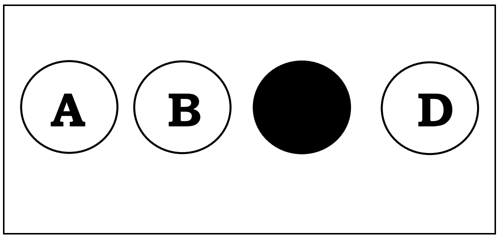
SUBJECT: Mathematics **TIME: 12:30 p.m. to 1:30 p.m.**

DAY: Tuesday **DATE: 5th July, 2022**

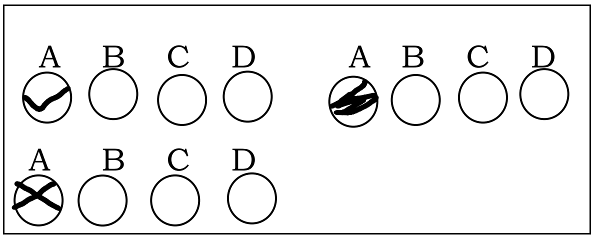
Important Instructions:

1. This test booklet is to be opened only when instructed by the invigilators to do so.
2. This booklet carries **50** questions in **7** printed pages. All carry equal marks.
3. For every correct answer, candidate will earn **2 marks**, for every wrong answer **0.5 marks** will be deducted.
4. Test Registration Number must be entered correctly in the OMR answer sheet, as advised by the invigilators. The Question Booklet code (A/B/C/D) must also be mentioned on the OMR answer sheet (if not printed already) as instructed.
5. Answers must be marked in the OMR answer sheet using a black or dark blue ball point pen only. The circle should be filled in completely, leaving no gaps.
6. Gadgets (Mobile phones, pagers, ear phones, music players, calculators, smart watches, etc.) are strictly prohibited in the exam hall. If any candidate is found in possession of any of these at his/her exam seat, he/she is liable to be disqualified.
7. In case of tie in the marks the merit will be considered based on total marks in qualifying examination.

Correct way of marking answer



Incorrect way of marking answer



Invigilator's Signature: __________________

A1

1. The space $\mathcal{C}^{(m)}[0,1]$ is a proper subspace of $\mathcal{C}^{(n)}[0,1]$ if and only if

(A) $m < n$. **(B)** $m = n$. **(C)** $m > n$. **(D)** $m \neq n$.

2. The span of the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and the plane $4y - 3z = 0$ in V_3 is

(A) the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$. **(B)** the plane $4y - 3z = 0$.

(C) the set $\{(0,0,0)\}$. **(D)** V_3.

3. The set $\{x, |x|\}$ is linearly dependent in the vector space

(A) $\mathcal{C}(-2,1)$. **(B)** $\mathcal{C}(-1,2)$. **(C)** $\mathcal{C}(-3,0)$. **(D)** $\mathcal{C}(-5,3)$.

4. If u and v are two nonzero vectors in a real inner product space V, then the number $\frac{u \cdot v}{\|u\|\|v\|}$ always lie in the interval

(A) $[0,1]$. **(B)** $[-1,\ 0]$. **(C)** $\left[-\frac{1}{2},\ \frac{1}{2}\right]$. **(D)** $[-1,\ 1]$.

5. There is/are ____________________ linear transformation(s) T from V_2 to V_2 satisfying $T(1,0) = (2,0)$ and $T(2,0) = (4,0)$.

(A) no **(B)** exactly one **(C)** exactly two **(D)** infinitely many

6. There is no onto linear transformation from V_n to V_m if

(A) $n < m$. **(B)** $n = m$. **(C)** $n > m$. **(D)** $n \neq m$.

7. Let $m, n \in \mathbb{N}$ and $T: V_n \to V_m$ be a linear map. Then which of the following is not necessarily true?

(A) $n(T) \leq m$ **(B)** $n(T) \leq n$ **(C)** $r(T) \leq n$ **(D)** $r(T) \leq m$

8. The sequence $\left\{\frac{n^{500}}{(1.005)^n}\right\}$

(A) converges to -1. **(B)** converges to 1.

(C) converges to 0. **(D)** diverges.

9. The series $\sum_{n=1}^{\infty}(-1)^n \frac{2n+1}{5n+3}$ is

(A) not alternating. **(B)** alternating but not convergent.

(C) convergent but not absolutely convergent. **(D)** absolutely convergent.

10. Let $f: [0,1] \to \mathbb{R}$. In which of the following case, f must be Riemann integrable over $[0,1]$?

(A) f^2 is Riemann integrable $[0,1]$. **(B)** f is bounded on $[0,1]$.

(C) f^4 is Riemann integrable $[0,1]$. **(D)** f^3 is Riemann integrable $[0,1]$.

A2

11. If $f(x) = \begin{cases} x, & x \in [0,1) \\ 1, & x \in [1,2] \end{cases}$, then f is

(A) Riemann integrable but not continuous on $[0,2]$.

(B) continuous but not Riemann integrable on $[0,2]$.

(C) Riemann integrable as well as continuous on $[0,2]$.

(D) neither Riemann integrable nor continuous on $[0,2]$.

12. Which of the following is FALSE for Fejer kernels K_n?

(A) $\frac{1}{2\pi}\int_{-\pi}^{\pi} K_n(x)dx = 1$.

(B) $K_n(-x) = K_n(x)$, $x \in \mathbb{R}$.

(C) $K_n(-x) = -K_n(x)$, $x \in \mathbb{R}$.

(D) $K_n(x) = \frac{1}{n+1}\sum_{m=0}^{n} D_m(x)$, $x \in \mathbb{R}$.

13. The Fourier series of the 2π-periodic function $f(x) = 1$, $x \in [0,2\pi]$, is

(A) 1.

(B) $\sum_{n=1}^{\infty} e^{inx}$.

(C) $\sum_{n=-\infty}^{\infty} e^{inx}$.

(D) $1 + e^{ix} + e^{i2x}$.

14. Let $f_n(x) = \frac{nx}{e^{nx}}$, $x \in [0,1]$, $n \in \mathbb{N}$. Then

(A) $\{f_n\}$ is divergent.

(B) $\{f_n\}$ converges uniformly on $[0,1]$.

(C) f_{1000} is not bounded.

(D) $\{f_n\}$ converges pointwise on $[0,1]$.

15. The rank of product AB of two $n \times n$ matrices A, B, is related with the rank of A and rank of B, in the form __________ .

(A) Rank of (AB) = rank of A + rank of B

(B) Rank of (AB) ≤ min. (rank of A, rank of B)

(C) Rank of (AB) ≤ max. (rank of A, rank of B)

(D) Rank of (AB) = |rank of A - rank of B|

16. If identity matrix I and a matrix A are square matrices of same order such that $I + A$ is invertible, then $(I + A)$ and $(\boldsymbol{I} + A)^{-1}$

(A) are same matrices.

(B) will commute.

(C) will not be commutative.

(D) are not conformable for product.

17. Consider a consistent system of three linear equations in three unknowns. If the rank of the coefficient matrix is 2, then the system has

(A) only one linearly independent solution.

(B) two linearly independent solutions.

(C) three linearly independent solutions

(D) five linearly independent solutions.

18. Diagonalized form D of a real symmetric matrix A

(A) will be inverse of A.

(B) will have entry 1 at the diagonal places.

(C) is not necessarily a symmetric matrix.

(D) will have eigen values of A on the diagonal places.

A3

19. Among the methods of finding a root of an equation in one variable, which method always converges?

(A) Newton-Raphson **(B)** Regila-falsi

(C) Graeffe's root squaring **(D)** bisection

20. Newton-Raphson's method for solving the equation $f(x) = 0$, is applicable if

(A) $|f'(x)| < |f(x)|$. **(B)** $f'(x)$ is non zero near a root.

(C) $|f'(x)| < |f''(x)|$. **(D)** $f'(x)$ is small near a root.

21. Lagrange's interpolation formula is useful for the data of

(A) equidistant and increasing values only.

(B) non equidistant, decreasing values only.

(C) both equidistant and non equidistant values.

(D) equidistant values only.

22. Weddle's rule for numerical integration will provide exact value of integral of $f(x)$, if $f(x)$ is

(A) an exponential function. **(B)** a quadratic polynomial.

(C) a trigonometric function. **(D)** a hyperbolic function.

23. Consider the metric space $(\mathbb{R}, d^*)$, where $d^*(x, y) = \frac{2|x-y|}{1+|x-y|}, \forall x, y \in \mathbb{R}$. Then $S_{\frac{3}{2}}(0)$, neighborhood of 0 with radius $\frac{3}{2}$ is

(A) $\mathbb{R}$. **(B)** $\left(-\frac{3}{2}, \frac{3}{2}\right)$. **(C)** $(-3, 3)$. **(D)** $\left(-\frac{3}{5}, \frac{3}{5}\right)$.

24. Consider the metric space $(\mathbb{R}, d)$, where $d(x, y) = |x| + |y|$ if $x \neq y$; and $d(x, y) = 0$ if $x = y$. Then the metric space $(\mathbb{R}, d)$ is

(A) a complete and compact. **(B)** a complete but not a compact.

(C) a compact but not complete. **(D)** neither compact nor complete.

25. Let E be the set of sequential limits of a sequence $\{a_n\}$ in a metric space X. Then E is always

(A) an open set. **(B)** a closed set. **(C)** a perfect set. **(D)** a compact set.

A4

26. Let $\{a_n\}$ be a sequence in a metric space (X, d). Consider the following statements:

(I) $\{a_n\}$ is a Cauchy sequence in (X, d);

(II) $\{d(a_{n+2}, a_n)\}$ converges to 0.

Which of the following statement is true?

(A) (I) implies (II) and (II) implies (I).

(B) (II) implies (I) but (I) need not imply (II).

(C) (I) implies (II) but (II) need not imply (I).

(D) neither (I) imply (II) nor (II) imply (I).

27. Let f be a continuous bijective map from a metric space X into a metric space Y. Then its inverse is continuous if

(A) Y is connected. **(B)** Y is compact.

(C) X is complete. **(D)** X is compact.

28. Let $E = \left\{\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$. Then the diameter of the set E is

(A) $\frac{2}{3}$. **(B)** $\frac{1}{2}$. **(C)** $\frac{3}{2}$. **(D)** 1.

29. The distance from the point $(-1,1)$ to the subspace $Y = \{(x, x) : x \in \mathbb{R}\}$ in the metric space $(\mathbb{R}^2, d_1)$, where $d_1\big((x_1, y_1), (x_2, y_2)\big) = |x_1 - x_2| + |y_1 - y_2|$ for $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2$, is

(A) 0. **(B)** $\sqrt{2}$. **(C)** $\frac{1}{\sqrt{2}}$. **(D)** 2.

30. Let G be a group which has exactly 36 distinct elements of order 7. Then the number of distinct subgroups of order 7 in G are ______________.

(A) 7 **(B)** 6 **(C)** 5 **(D)** 4

31. There are ___________ group homomorphism $\mathbb{Z}_{28}$ onto $\mathbb{Z}_6$. Here $\mathbb{Z}_m$ denote the group integers modulo m under addition $+_m$.

(A) zero **(B)** 28 **(C)** 6 **(D)** 2

32. Let G be a group with $o(G) = 1147$.

(A) Then G is cyclic and all p-Sylow subgroups of G are normal.

(B) Then G is cyclic and all p-Sylow subgroup of G are not normal.

(C) Then G is non – cyclic and all p-Sylow subgroups of G are normal.

(D) Then G is non – cyclic and all p-Sylow subgroups of G are not normal.

A5

33. Let $\mathbb{Z}_m$ denote the group integers modulo m under addition $+_m$. Then the direct product of $\mathbb{Z}_{10}$ and $\mathbb{Z}_{19}$ i.e $\mathbb{Z}_{10} \times \mathbb{Z}_{19}$. will

(A) have cyclic subgroup of orders 10, 19, 190.

(B) have cyclic subgroup of orders only order 10 and 19.

(C) have cyclic subgroup of orders only 19 and 190.

(D) have cyclic subgroup of orders only 10 and 190.

34. Number of maximal ideals in the ring $(\mathbb{Z}_{60}, +_{60}, \cdot_{60})$ are ___________.

(A) 60 **(B)** zero **(C)** 5 **(D)** 3

35. Consider the additive group of rational numbers, $\mathbb{Q}$. Suppose $f : \mathbb{Q} \to \mathbb{Q}$ is a homomorphism.

(A) Then f is always a zero homomorphism.

(B) Then f is either a zero homomorphism or an isomorphism.

(C) Then f is always given by $f(t) = \alpha t$, $t \in \mathbb{Q}$, for some non – zero real number α.

(D) None of the above holds.

36. Let R be a commutative ring with unit element and let $f : R \to R$ be defined by $f(t) = t^2$.

(A) Then f is always a ring homomorphism.

(B) Then f is a ring homomorphism only if characteristic of R is 2.

(C) Then f is a ring homomorphism only if characteristic of R is a prime p, $p > 2$.

(D) Then f is a ring homomorphism only if characteristic of R is 0.

37. Value of $\left[i^{26} + \left(\frac{1}{i}\right)^{21}\right]^2$ is

(A) $2i$. **(B)** $-1 - i$. **(C)** i. **(D)** $1 + i$.

38. For a complex number z, the number of solutions of the equation $z^2 + |z|^2 = 2 + 2i$ is / are

(A) 4. **(B)** zero. **(C)** 2. **(D)** infinitely many.

39. If α and β are roots of the equation $2x^2 - 7x + 8 = 0$, then the equation whose roots are $4\alpha + 5\beta$ and $5\alpha + 4\beta$ is

(A) $2x^2 - 63x - 498 = 0$. **(B)** $2x^2 - 63x + 498 = 0$.

(C) $-2x^2 - 63x + 498 = 0$. **(D)** $2x^2 - 498x + 63 = 0$.

A6

40. Consider two equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$. For $a \neq b$, suppose both the equations have a common root. Then $a + b =$

(A) 1. **(B)** 0. **(C)** – 2. **(D)** – 1.

41. The solution of the initial value problem $\frac{dy}{dx} + y = f(x)$, where $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$ and $y(0) = 0$ is ________________.

(A) $y(x) = \begin{cases} 2(e^{-x} - 1), & 0 \le x \le 1 \\ 2e^{-x}(e-1), & x > 1 \end{cases}$ **(B)** $y(x) = \begin{cases} 2(1 - e^{-x}), & 0 \le x \le 1 \\ 2e^{-x}(e-1), & x > 1 \end{cases}$

(C) $y(x) = \begin{cases} 2(1 - e^{-x}), & 0 \le x \le 1 \\ 2e^{-x}(1-e), & x > 1 \end{cases}$ **(D)** $y(x) = \begin{cases} x - 1 + e^{-x}, & 0 \le x \le 1 \\ e^{-x}, & x > 1 \end{cases}$

42. The solution of the equation $y'' + ay' + by = 0$, where a and b are constants, approaches to zero as $x \to \infty$, then

(A) $a > 0,\ b > 0$. **(B)** $a > 0, b < 0$.

(C) $a < 0, b > 0$. **(D)** $a < 0, b < 0$.

43. The singular solution of the differential equation $y = x\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right)^2$ represents

(A) a parabola passing through the origin .

(B) a parabola passing through $(1,1)$.

(C) a straight line passing through the origin.

(D) a straight line passing through $(1, 1)$.

44. Let y_1 and y_2 be two linearly independent solutions of the differential equation $xy'' - 2x^2y' + e^x y = 0$, satisfying $y_1(0) = 1, y_2(0) = -1, y_1'(0) = 1, y_2'(0) = 1$. Then the Wronskian of y_1 and y_2 at $x = 2$ is

(A) $2e^{-4}$. **(B)** $2e^{-1}$. **(C)** $2e^{4}$. **(D)** $2e^{2}$.

45. The solution of the PDE $zp = -x$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, is

(A) $x^2 + y^2 = f(z)$. **(B)** $x^2 + z^2 = f(y)$.

(C) $y^2 + z^2 = f(x)$. **(D)** $x^2 + y^2 = f(xyz)$.

A7

46. Which of the following statements is true for the function

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases} \ ?$$

(A) $f_x(0,0), f_y(0,0)$ exist and f is differentiable at $(0,0)$.

(B) $f_x(0,0), f_y(0,0)$ exist and f is not differentiable at $(0,0)$.

(C) $f_x(0,0), f_y(0,0)$ do not exist and f is differentiable at $(0,0)$.

(D) $f_x(0,0), f_y(0,0)$ do not exist and f is not differentiable at $(0,0)$.

47. The slope of the tangent to the curve of intersection of the paraboloid $z = 16 - 4x^2 - y^2$ and the plane $y = 2$ at the point $(1,2,8)$ is

(A) -4. **(B)** -8. **(C)** 0. **(D)** 1 .

48. The temperature T at a point (x,y,z) is given by $T(x,y,z) = 200\ e^{-x^2-3y^2-9z^2}$, where T is temperature measured in $°C$ and x, y, z in meters. Then the direction in which temperature increases fastest at the point $(2,-1,2)$ is

(A) $-2\boldsymbol{i} - 3\boldsymbol{j} - 18\boldsymbol{k}$. **(B)** $-2\boldsymbol{i} + 3\boldsymbol{j} - 9\boldsymbol{k}$.

(C) $-2\boldsymbol{i} + 3\boldsymbol{j} + 9\boldsymbol{k}$. **(D)** $-2\boldsymbol{i} + 3\boldsymbol{j} - 18\boldsymbol{k}$.

49. The area $A(S)$ of the part of the plane $z = ax + by + c$ that project on to a region D in the xy-plane with area $A(D)$ is

(A) $A(S) = abc\ A(D)$. **(B)** $A(S) = (a+b+c)\ A(D)$.

(C) $A(S) = \sqrt{a^2+b^2+1}\ A(D)$. **(D)** $A(S) = \sqrt{a^2+b^2}\ A(D)$.

50. The work done by the force field $\boldsymbol{F}(x,y,z) = xz\boldsymbol{i} + xy\boldsymbol{j} + yz\boldsymbol{k}$ on a particle that moves along the curve $\boldsymbol{r}(t) = t^2\boldsymbol{i} - t^3\boldsymbol{j} + t^4\boldsymbol{k}, 0 \leq t \leq 1$ is

(A) $\frac{88}{23}$. **(B)** $\frac{55}{88}$. **(C)** $\frac{32}{88}$. **(D)** $\frac{23}{88}$.

ALL THE BEST

GENERAL INSTRUCTIONS FOR ADMISSION

1. Candidate is required to attach self –attested copies of the following documents with his / her copy of the downloaded application form:
   (A) B. Sc. Semester – 5 and Semester – 6 marksheets (if in semester system).
   (B) F. Y. B. Sc., S. Y. B. Sc. and T. Y. B. Sc. marksheets (if in annual system).
   (C) Certificate for SC / ST/ SEBC /EWS candidates.
   (D) Valid non – creamy layer certificate (if SEBC candidate).
   (E) Eligibility certificate for students with B. Sc. other than The M. S. University students.

2. In case original marksheet of the last examination is not issued by the University, the print out of the marksheet on website maybe attached.

3. B. A. Mathematics students of The M. S. University of Baroda are required to attach the transfer certificate.

4. Students who have done their B. Sc. from a university outside Gujarat must submit syllabi of all semester duly signed by Head / Principal of respective department / college. This copy is to be submitted to the office of the Department of Mathematics, Faculty of Science, The M. S. University of Baroda.

Key for Set A

1	**C**	21	**C**	41	**D**
2	**B**	22	**B**	42	**A**
3	**C**	23	**C**	43	**A**
4	**D**	24	**B**	44	**C**
5	**D**	25	**B**	45	**B**
6	**A**	26	**C**	46	**B**
7	**A**	27	**D**	47	**B**
8	**C**	28	**A**	48	**A**
9	**B**	29	**D**	49	**C**
10	**D**	30	**B**	50	**D**
11	**C**	31	**A**		
12	**C**	32	**A**		
13	**A**	33	**A**		
14	**D**	34	**D**		
15	**B**	35	**B**		
16	**B**	36	**B**		
17	**A**	37	**A**		
18	**D**	38	**C**		
19	**D**	39	**B**		
20	**B**	40	**D**		